HEAT TRANSFER ACCOMPANYING FORCED CONVECTION IN A POORLY PERMEABLE MEDIUM

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Heat transfer accompanying liquid motion in a medium of low porosity is investigated.

In connection with the exploitation of geothermal energy, it is very important to investigate the heat transfer accompanying the motion of water through underground permeable beds — collectors.

The physical velocity $\vec{w}$ of liquid motion in an artificial collector is determined by the permeability ξ (darcy) characterizing the active porosity of the bed. It is evident from Fig. 1, which presents the known data on the permeability of artificial collectors, that the collector permeability (and hence the porosity) may vary by several orders of magnitude.

Consider the heat transfer accompanying forced convection in a weakly permeable collector, assuming that the bed is filled by hot liquid (temperature t_b^I) and the bed skeleton is formed by spherical structures. In the one-dimensional case, for an average liquid temperature t_l over the cross section, the energy equation is

$$G_l \frac{\partial t_l}{\partial \tau} + G_l w_x \frac{\partial t_l}{\partial x} = \sigma q_v (t_l), \tag{1}$$

where q_v is the heat flux from the skeleton passing through specific surface σ; w_x is the true velocity of liquid motion in the collector: $w_x = Q/mS$.

In what follows, the position developed in [1, 2] is adopted. The heat flux from the bed skeleton $q_v(t_l)$ is found by solving the heat-conduction equation for a single sphere:

$$\frac{\partial}{\partial \mathrm{Fo}} (RT) = \frac{\partial^2}{\partial R^2} (RT), \quad 0 < R < 1 , \tag{2}$$

with boundary conditions of the third kind at the surface:

$$-\left.\frac{\partial T}{\partial R}\right|_{R=1} = \mathrm{Bi}\,(T - 1)|_{R=1}. \tag{3}$$

In Eq. (3) the temperature of the liquid phase is taken equal to 1: In this formulation, the thermal resistance from the direction of the liquid can be taken into account. The solution of Eq. (2) with the boundary conditions (3), zero initial conditions, and symmetry condition $T_R'(R = 0) = 0$ is known [3]. If the liquid-phase temperature varies arbitrarily, according to a law $\Theta(F_o)$, the desired temperature is equal to the product of representations of the temperature (Duhamel theorem). The resulting temperature is written in Laplace–Carson transforms:

$$T(\mathrm{Fo}) \div\!\rightarrow \bar{T}(s) = \frac{\mathrm{Bi}\,\mathrm{sh}(\sqrt{s}R)}{R\,[(\mathrm{Bi} - 1)\,\mathrm{sh}\sqrt{s} + \sqrt{s}\,\mathrm{ch}\sqrt{s}]} \bar{\Theta}(s). \tag{4}$$

The Laplace–Carson representation of the heat flux is

$$\bar{q}_v(s) = -\frac{\lambda_b (t_0^{in} - t_b^I)}{r_0} \cdot \left.\frac{\partial T}{\partial R}\right|_{R=1} = -\frac{\lambda_b (t_0^{in} - t_b^I)}{r_0} \times \frac{(\sqrt{s} - \mathrm{th}\sqrt{s})\,\bar{\Theta}(s)}{(1 - B)\,\mathrm{th}\sqrt{s} + B\sqrt{s}}, \tag{5}$$

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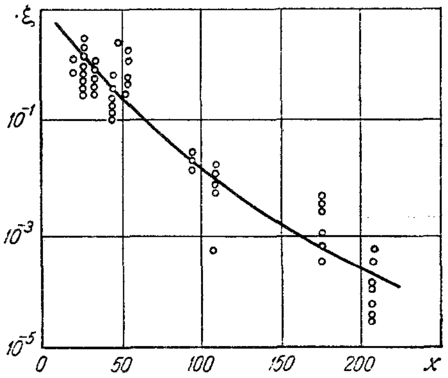

Fig. 1. Permeability ξ (darcy) of artificial collectors. x, m.

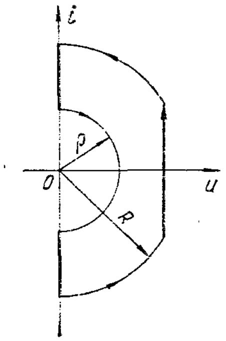

Fig. 2. Integration loop.

where B ≡ 1/Bi. Let

$$g\,(\mathrm{Fo}) \div\!\!\to \bar{g}\,(s) \equiv -\frac{\sqrt{s} - \mathrm{th}\sqrt{s}}{(1-B)\,\mathrm{th}\sqrt{s} + B\sqrt{s}}\,.$$

After conversion to a dimensionless representation, Eq. (1) may be rewritten in integro-differential form

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} + \frac{\partial \Theta}{\partial X} = G\,\frac{\partial}{\partial \mathrm{Fo}} \int\limits_0^{\mathrm{Fo}} g\,(u)\,\Theta\,(\mathrm{Fo} - u)\,du. \tag{6}$$

Applying a two-dimensional Laplace–Carson transform to Eq. (6) with initial and inlet conditions

$$\Theta\,(X,\ \mathrm{Fo}=0)=0;\ \ \Theta\,(X=0,\ \mathrm{Fo})=1, \tag{7}$$

we obtain an equation in the representations

$$s\bar{\Theta} + p\bar{\Theta} - p = \bar{g}\,(s)\,\bar{\Theta}. \tag{8}$$

Finding $\bar{\Theta}(s)$ from Eq. (8), the general formula of operational calculus [3] gives

$$\Theta\,(\mathrm{Fo}) = \frac{1}{2\pi i}\int\limits_{\gamma - i\infty}^{\gamma + i\infty} \exp\,[-sX]\,\exp\left[s\mathrm{Fo} - GX\,\frac{\sqrt{s} - \mathrm{th}\sqrt{s}}{(1-B)\,\mathrm{th}\sqrt{s} + B\sqrt{s}}\right]\frac{ds}{s}\,. \tag{9}$$

The integration loop is chosen in accordance with Fig. 2, avoiding the zero point, and then the Jordan lemma gives the final form of the solution:

$$\Theta\,(\mathrm{Fu},\ \mathrm{Bi}) = \frac{1}{2} + \frac{2}{\pi}\int\limits_0^{\infty} \exp\,[-GX\varphi_1(u)]\,\sin\left[\frac{u^2\mathrm{Fu}}{2} + GX\varphi_2(u)\right]\frac{du}{u}\,, \tag{10}$$

where

$$\varphi_1(u) = (u\,n_1 - 2\,(1-B)\,n_2 + u^2B)/\beta;\ \ \beta = 2\,(B - B^2)\,n_1u + 2\,(1-B)^2\,n_2 + u^2B;$$

$$\varphi_2(u) = u\ n_3/\beta;\ \ \mathrm{Fu} = \mathrm{Fo} - X;\ \ n_1 = (\mathrm{sh}\,u + \sin u)/(\mathrm{ch}\,u + \cos u);$$

$$n_2 = (\mathrm{ch}\,u - \cos u)/(\mathrm{ch}\,u + \cos u);\ \ n_3 = (\sin u - \mathrm{sh}\,u)/(\mathrm{ch}\,u + \cos u).$$

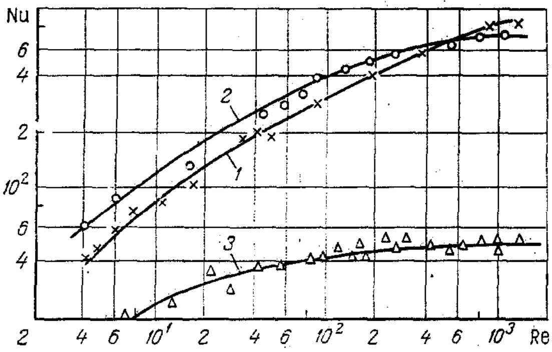

Fig. 3. Dependence of heat-transfer coefficient Nu on Re (Sc = 2500): 1) glass spheres (m = 0.4); 2) particles of irregular form (m = 0.27); 3) rectangular parallelepipeds (m = 0.05).

Calculation by Eq. (10) is possible when the value of Bi corresponding to the type of filtering medium and the active porosity is known.

To this end, an experimental investigation of the heat-transfer coefficient for various models of a porous medium was carried out by the electrochemical-analogy method [4] at the Institute of Catalysis of the Siberian Branch of the Academy of Sciences of the USSR (É. Brandes assisted in the work).

The working part of the experimental apparatus was a sectional cylindrical tube of organic glass (diameter 200 mm), which was filled with material in the experiments. The permeability of the medium was modeled by glass particles of irregular form (d_{eq} = 11.5 mm; active porosity m = 0.3) and rectangular parallelepipeds (3.5 × 3.5 × 150 mm; m = $5 \cdot 10^{-2}$). Electrochemical sensors in the form of the filling elements recorded the limiting value of the diffusion current.

The flow conditions were varied so that Re covered the range 3-1400. From the results of the measurements, curves of Nu = f(Re, Sc) were plotted (Fig. 3). The calibration curve 1 was obtained for glass spheres (d = 1.8 mm; m = 0.4); the results were in good agreement with those of similar investigations [4].

It is evident from Fig. 3 that decrease in active porosity by an order of magnitude leads to a sharp fall in the heat-transfer coefficient α.

For mean values of the porosity (m ∿ 0.5-0.2), α may be calculated from the formulas

$$\mathrm{Nu} = \begin{cases} 10.5\,\mathrm{Pr}^{0.33}\mathrm{Re}^{0.6}, & \mathrm{Re} < 35, \\ 9.2\,\mathrm{Pr}^{0.33}\mathrm{Re}^{0.6}, & 35 \leqslant \mathrm{Re} < 140, \\ 10.2\,\mathrm{Pr}^{0.33}\mathrm{Re}^{0.6}, & \mathrm{Re} > 140, \end{cases} \tag{11}$$

which are given in [4] and recalculated for Pr.

For low porosity the experimental results are treated by the method of least squares, giving

$$\mathrm{Nu} = \begin{cases} 7.8\,\mathrm{Pr}^{0.33}\mathrm{Re}^{0.20}, & \mathrm{Re} \leqslant 140, \\ 15.8\,\mathrm{Pr}^{0.33}\mathrm{Re}^{0.06}, & \mathrm{Re} > 140. \end{cases} \tag{12}$$

The data on the heat-transfer coefficient — Eq. (12) — may be used to determine the range of the Biot number: Bi = 0.5-200.

The estimates of [5] show that in an underground collector of a circulation system, usually, Re > 100 and Bi > 50; therefore, Eq. (10) may be significantly simplified by passing to the limit (B = 0) Bi → ∞, to give

$$\Theta(\mathrm{Fu},\ GX) = \frac{1}{2} + \frac{2}{\pi}\int\limits_0^\infty \exp\left[-GX\,\varphi_1(u)\right] \sin\left[\frac{u^2\,\mathrm{Fu}}{2} + GX\varphi_2(u)\right]\frac{du}{u}\,, \tag{13}$$

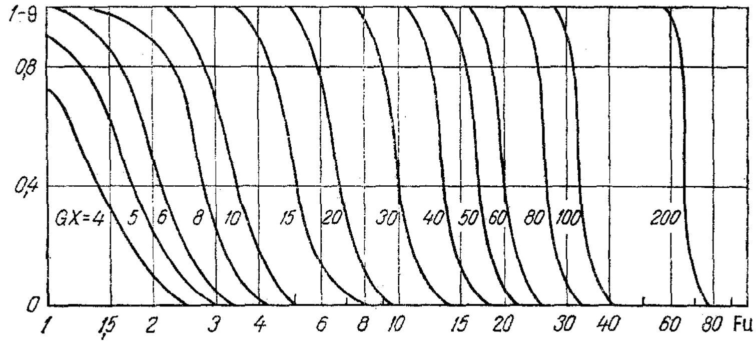

Fig. 4. Dimensionless temperature according to Eq. (13).

where

$$\varphi_1(u) = un_1/(2n_2);\ \ \varphi_2(u) = un_3/(2n_2).$$

Tabular values of the dimensionless temperature $\varkappa$(Fu, GX) given by Eq. (13) are shown in Fig. 4. For a relatively crack-free medium the specific surface $\sigma(m^2/m^3)$ is given by the formula [5]

$$\sigma = k(1-m)/r_0, \tag{14}$$

in which k = 3 if the skeleton consists of spherical structures (k = 2 for parallel cracks).

For small Re, calculations should be based on the general formula (10), finding Bi from Fig. 3.

NOTATION

m, porosity (fractions of unity); τ, time; w_x, true velocity of liquid motion along x axis; r_0, radius of spherical structures; r, radius; a_i, λ_i, thermal resistivity and thermal conductivity of i-th material; α, heat-transfer coefficient; ν, kinematic viscosity; d_{eq}, equivalent particle diameter; Q, volume flow rate of liquid, m^3/sec; S, bed cross-sectional area; σ, specific surface of bed skeleton, 1/m; k, form factor; t_i, G_i, temperature and bulk heat capacity of i-th material; t_0^{in}, temperature at collector inlet; s, p, parameters of Laplace–Carson transform. Dimensionless parameters: liquid-phase temperature $\Theta = (t_l - t_b^I)/(t_0^{in} - t_b^I)$; temperature of bed skeleton $T = (t_b - t_b^I)/(t_0^{in} - t_b^I)$; $Fo = a_b\tau/r_0^2$; $X = a_b x/w_x r_0^2$; $G = G_b\sigma r_0/G_l$; $R = r/r_0$; $Pr = \nu/a_l$; $Re = w_x d_{eq}/\nu$; $Bi = \alpha r_0/\lambda_b$; $Nu = \alpha d_{eq}/\lambda_l$; Sc, Schmidt number. Indices: l, liquid; b, bed.

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